Geodesy and interior structure of Mercury

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1 Tidal potential
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Tidal potential (I)

- tidal force = differential gravitational force
- gradient of a tidal potential
  - direct effect of the Sun
  \[
  V_T = -\frac{GM\odot}{d} \sum_{l=2}^{\infty} \left( \frac{r}{d} \right)^l P_l(\cos \Psi)
  \]
  - orbital motion (Kepler’s laws)
  - rotational motion
  - restrict to degree 2
- Venus: $4 \times 10^{-6}$ smaller
- indirect effect due to planetary effects on orbital motion
- VSOP87 ephemerides (Bretagnon & Francou 1988) valid for several thousand years around J2000.0
Tidal potential (II)

- Tidal deformation and potential can mathematically be described with three spherical harmonics.
- Main period: half a Mercury solar day = one Mercury year (3:2 resonance).
- No simple division as for the Earth: typical periods of zonal, tesseral, and sectorial waves are long period, diurnal, and semidiurnal.

**Figure:** sectorial waves
Tidal potential (III)

**Figure:** tesseral waves

**Figure:** zonal waves
Tidal reaction

Tidal potential causes

- periodically varying surface displacements (Love numbers $h$ and $l$)
  - estimate for equipotential surface: $\frac{V_T}{g} \approx 1\text{m}$
  - $\delta r = h \frac{V_T}{g}$

- variations in the external potential field (Love number $k$)
  - estimate: $\frac{V_T}{V} \approx \frac{M_\odot}{M} \left( \frac{R}{a} \right)^3 \approx 5 \times 10^{-7}$
  - $\delta V = (1 + k) V_T$ (at surface)

- surface gravity variations (Love numbers $h$ and $k$)
  - estimate: gradient of potential: $2 \frac{V_T}{R} = 3 \times 10^{-6}\text{ms}^{-2}$
  - $g = 3.7\text{ms}^{-2}$
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2 **INTERIOR STRUCTURE MODELS**
   - Composition
   - Core modeling

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Basic facts

- mass $M = 3.302 \times 10^{23}$ kg (Anderson et al. 1987)
- radius $R = 2439 \pm 1$ km
- density $\rho = 5430 \pm 10$ kg/m$^3$
- large core
Composition (I)

- **data**
  - large Fe/Si ratio (large core)
  - low surface FeO content (spectral observations)

- **mantle**
  - mantle mineralogy: assume olivine, pyroxene, garnet
  - chemical composition: strongly dependent on formation history (Taylor and Scott 2005)
  - density, rigidity and incompressibility: relatively small differences
  - density variation $\approx 100 \text{ kg/m}^3$ (few %)

- **mantle model**
  - homogeneous density $\rho = 3500 \text{ kg/m}^3$
  - rigidity and incompressibility: same pressure dependence as in upper mantle of the Earth
Composition (II)

core:
- Fe + S (abundant + soluble at Mercury pressures)
- $x_S$ between 0.1 wt% and 14 wt%
- density, rheological parameters corrected for $P$ and $T$
- $\gamma$-iron: fcc phase

FeS phase diagram (Fei et al. 1995)
Core evolution (I)

- models ranging from entirely liquid to entirely solid core
- $\delta\rho$ between solid and liquid $\approx 3.5\%$ (Anderson 2003)
- relatively low pressure compared to Earth: sulfur almost doesn’t solidify with iron ($x_S < x_S^{eut}$)
- pure iron inner core

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<th>EXP ID</th>
<th>$P$ (GPa)</th>
<th>$T$ (K)</th>
<th>Time (min)</th>
<th>$S_{\text{solid}}$ (at.%$)$</th>
<th>$S_{\text{liquid}}$ (at.%$)$</th>
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**Figure:** sulfur solidification (Li et al. 2001)
Core modeling (II)

- almost pure iron core, increasing sulfur concentration in outer core
Phase diagrams

- maximum S concentration: eutectic composition \( (x_S \approx 22\%) \)

**Figure:** melting in Fe-FeS (Fei et al. 1997, 2000)

- eutectic reached for large inner core, low pressure
- Ni increases sulfur content of eutectic composition
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Tidal displacements

4 wt% sulfur, inner core radius=1000 km. Starting from J2000.0

radial (solid line), East-West (dashed) and North-South (dotted) displacements at the equator

solid line: equator (sect.+zonal), dashed line: 30° latitude, dotted line: 60°, dashed-dotted line: 90° (zonal)
EXTERNAL POTENTIAL AND GRAVITY VARIATIONS

Starting from J2000.0

at 85° latitude
MORE accuracy for degree-two: \(10^{-9}\) (Milani et al. 2001)

at equator
• solid cores: 5 times smaller
• Love numbers increase with
  • increasing core radius
  • decreasing sulfur concentration in outer core
• MORE accuracy: $\lesssim 1\%$ (Milani et al. 2001)
• important constraint on core: strong reduction of possible models
**Love numbers $h$**

- measurements: laser altimeter + radio tracking of orbiter
- BELA simulations (Christensen et al. EGU2006): accuracy of a few % on $h_2$
- much stronger constraint on interior form combination of both Love numbers: error on sulfur concentration of a few percent, and on core radius some ten of kilometers
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Forced libration

\[ \Delta \varphi = \frac{3}{2} \frac{B-A}{C_m} \left( 1 - 11e^2 + \frac{959}{48}e^4 + \ldots \right) \]

- amplitude liquid core \( \approx 2 \times (\text{amplitude solid core}) \)
- for our models: core size is determining factor
- effect of core-mantle coupling < 1%
- constraint on models
- even larger range: fixed \( B - A \) and fixed mantle assumed
GRAVITY FIELD DETERMINATION

- \(( \frac{B-A}{MR^2} ) = 4C_{22}\)
- presently badly known: 
  \(C_{22} = (1.0 \pm 0.5) \times 10^{-5}\) (Anderson et al. 1987)
- MESSENGER: precision below 1% (Solomon et al. 2001)
- MORE: 0.01% (Milani et al. 2001)
Libration

- Alternatively: free libration, but damped

\[ P_{\text{free}} = \frac{2\pi}{n} \left[ \frac{1}{3} \frac{C^m}{B-A} e^\left(\frac{7}{2} - \frac{123}{16} e^2\right) \right]^{1/2} \]

- solid core: 15.830 years
- liquid core: 10.5 years to 12 years (Rambaux et al. 2007)
- other forced libration periods easily separable (Peale et al. 2007)
Obliquity

\[
\frac{C}{MR^2} = \left[ \frac{J_2}{(1-e^2)^{3/2}} + eC_{22}\left(7 - \frac{123}{8} e^2\right) \right] \frac{n}{\mu} \left[ \frac{\sin l}{\epsilon_C} - \cos l \right]
\]

- The polar moment of inertia \( C \) can be determined from measuring the obliquity \( \epsilon_C \).
- Relation valid for Mercury occupying its Cassini state
- Theoretical range: about \([1, 2.5]\) arcmin
- Caveat: spin axis does not occupy Cassini state
  - free precession: expected damped
  - spin does not follow Cassini state due to planetary perturbations.
- Margot’s measurements (unpublished) seem to agree with the theoretical values and to confirm that Mercury occupies the Cassini state (Peale 2006, Yseboodt and Margot 2006).
Moments of inertia

- expected accuracy: 1 as, or 1% (RSDI, space missions)
- nominal BepiColombo minimum needed
- libration ($C_m$) less sensitive to inner core
- Peale 1976: \[
\left( \frac{C_m}{B-A} \right) \left( \frac{B-A}{MR^2} \right) \left( \frac{MR^2}{C} \right) = \frac{C_m}{C} \leq 1
\]
Ground-based observations

- Radar Speckle Displacement Interferometry (RSDI)
  - one-shot precision: 2 arcsec
  - long observation campaigns: 0.2 arcsec
- Margot et al. 2004 (AGU): $\Delta \varphi \approx 60 \pm 6$as, $\epsilon_C = 2.1 \pm 0.1$amin (but new values)
- Although there are still large uncertainties on $B - A$ and obliquity, this value shows that, with very high probability (95%), the core is liquid
- $J_2$ at the low end of the Mariner 10 values
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CONCLUSIONS

• main geodetic constraints on core:
  • tides: $k_2$, $h_2$, 1% precision, 25% uncertainty ($0.4 \pm 0.1$)
  • obliquity: $C$, 1% precision (1as), 4% uncertainty ($0.345 \pm 0.015$)
  • libration: $C_m/C$, 2.5% precision (1as), 15% uncertainty ($0.5 \pm 0.075$ for our models)

• tidal measurements maybe most important for core radius

• different sensitivities to the interior structure

• combination of measurements of the low-degree gravitational field, the rotation, and the tides of Mercury will improve our knowledge of Mercury’s interior