Noname manuscript No.
(will be inserted by the editor)

**Andrea Milani · Giovanni F. Gronchi ·
Zoran Knežević**

New Definition of Discovery for Solar System Objects

Received: 23 February 2006; Revised: 13 June 2006

Abstract We propose a new formal definition of discovery for a Solar System object. It is based on an objective and mathematically rigorous algorithm to assess when a set of observations is enough to constitute a discovery. When this definition is satisfied, in almost all cases the orbit is defined well enough to establish the nature of the object discovered (Main Belt vs. Near Earth Asteroid, Trans-Neptunian vs. long period comet). The frequency of occurrence of exceptions is estimated by a set of numerical experiments.

The availability of a non-subjective definition of discovery allows some rules to be adopted for the assignment of discovery credit with a minimum risk of dispute. Such rules should be fair, encourage good practice by the observers and acknowledge the contribution of the orbit computers providing the identifications and the orbits, as well as the one of all the contributing observers.

Keywords discovery · orbit determination · asteroids · comets · Trans-Neptunians

A. Milani
Department of Mathematics, University of Pisa, Italy
E-mail: milani@dm.unipi.it

G. F. Gronchi
Department of Mathematics, University of Pisa, Italy
E-mail: gronchi@dm.unipi.it

Z. Knežević
Astronomical Observatory, Belgrade, Serbia
E-mail: zoran@aob.bg.ac.yu

1 Purpose

This paper proposes a new definition of *discovery*, applicable to natural *moving objects* belonging to our Solar System. It also discusses how to attribute *discovery credit* and *naming rights*.

The principles used in the proposals contained in this paper are the following. The definition of discovery, and the attribution of credit, should depend only upon the information contained in the data and in the computations submitted to an IAU sponsored Data Center for publication and upon the date of submission. The algorithm to compute whether a given set of data and/or computations qualifies as a discovery must be public and reproducible: it should be available as free software.

The definition of discovery and the attribution of credit for it should not depend upon assumptions on what the observers and/or the orbit computers may or may not have known before submission. The data and computations submitted to the Data Center should be made public, in an essentially instantaneous way, thus all the actors have prompt access to them; they are assumed to use this information, and if they in fact have more information than the one which is published, this is not relevant for discovery credit.

The basic idea is that a Solar System object is considered *discovered* when enough data have been gathered and attributed to a single object, to allow its nature to be established. If a smaller dataset is accumulated by one or more observers, they should receive credit for their contribution if and when the dataset is incorporated in a larger one which can be considered a discovery, but they cannot have exclusive credit.

This paper is organized as follows. Section 2 provides the definitions of the entities used in the main definition of discovery, which is given in Section 3. Discovery credit and naming rights are discussed in Section 4. Section 5 describes the methods currently used for defining and crediting discoveries and outlines the need for change. Section 6 provides the mathematically rigorous definitions and computational algorithms. Section 7 contains results from numerical experiments of orbit determination with the goal of showing that the new rigorous definition of discovery fulfills, at least in the statistical sense, the qualitative requirement of establishing the nature of the discovered object, while a definition based only on the number of observed nights fails in critical cases, especially the ones most interesting for current research. Section 8 summarizes the conclusions and discusses some options for the parameters used in the definitions.

2 Definitions

2.1 Actors

In this subsection we define the people/organization involved in the procedure of discovery, and the objects whose discovery is discussed in this paper.

Definition: Observer Either a person or a team or a project or an astronomical observatory; in the context of this paper it is assumed that the

Observer is claiming credit for submitting some astrometric data and accepting that they become public. The submission may also contain photometric data. If more than one Observer are listed in the authors field, the number and order of the names is defined by the submission and cannot be contested by the Data Center.

Definition: Orbit Computer Either a person or a team or a project or a scientific institution; in the context of this paper it is assumed that the Orbit Computer is claiming credit for submitting the results of computations, containing orbit solutions, with uncertainty, and identifications between sets of data already submitted by the Observers[1]

Definition: IAU Data Center The organization (with a mandate from the IAU) in charge of receiving the data submitted by both Observers and Orbit Computers[2]. All the submissions have to be understood as requests for publication of the data; the Data Center time stamps the submissions and publishes them on Internet with the shortest technically possible delay, typically not more than a few seconds. The same Data Center may have also the task of processing the data, that is of acting as an Orbit Computer, and also of assigning Discovery Credit as discussed in Section 4, but these three functions have to be clearly separated. Another task for the Data Center is to assess the (astrometric, possibly also photometric) accuracy of the observations supplied by the different Observers under different conditions, with the goal of establishing a statistically reliable Error Model.

Definition: Solar System Object (SSO) Includes natural bodies orbiting in the Solar System: asteroids, comets, Trans-Neptunian objects, satellites of the major planets[3]. Some objects may belong to our Solar System only temporarily, as for hyperbolic comets. There is a problem in setting a minimum size: meteoroids with a few meters diameter have been observed. Artificial satellites should not be considered in this context: in most cases their observations can be easily discriminated. Interplanetary space probes (or spent rocket stages) sometimes return to the neighborhood of the Earth and are rediscovered: in such a case it may not be obvious whether they are natural or artificial.

This paper does not give the definitions for the discovery of natural satellites of planets/asteroids: the mathematical theory needs to be formulated in a different way. It is likely that an analogous definition of discovery could be built also for satellites, by similar arguments.

[1] The submission of astrometric data and Identifications/Orbits can be simultaneous; then it should indicate the authors of the different contributions.

[2] In the future, such organization may be called with the traditional name *Minor Planet Center*; here we are not using this name to keep the distinction with the present MPC, which is operating in a very different way.

[3] The observations of the major planets are processed in a separate way. This creates some problems due to the uncertain definition of major planet; e.g., the data of Pluto should be processed in the same way as the data of the other Trans-Neptunians.

2.2 Data

In this subsection we define the data and dataset to be combined into discoveries. In short, these are the data submitted by the Observers.

Definition: Observation (OBS) A set of data uniquely defining a position on the celestial sphere at a given time, two angles (right ascension, declination) and possibly an apparent magnitude. An Observation should always be provided with the meta-data necessary to assess the accuracy, including at least the instrument used, the signal to noise ratio and the star catalog used in the astrometric (and photometric) reduction, possibly also the standard deviation of time, angles and magnitude as estimated by the observer. This should not be confused with the a posteriori accuracy of the observation, as estimated by statistical quality control of the data supplied by a given observer and included in the Error Model.

Definition: Detection of a Moving Object (DMO) An Observation corresponding to a real moving object, which is a SSO. Note that in real cases (as opposed to simulations) we do not know which Observations are *false*, that is belonging to no real body, which belong to a fixed star and which are DMO, that is correspond to a real SSO. Only by identifying many detections as belonging to SSOs, and by fitting orbits, can we draw conclusions on some of the Observations, by no means on all. If the data are collected by a survey with the purpose of discovering as many SSO as possible, the fraction of false Observations needs to be large, such as 50%, to avoid losing dim discoveries; in these cases the distinction between OBS and DMO is very important. This also implies that an individual OBS cannot be submitted as a DMO[4].

Definition: Very Short Arc (VSA) A number of OBS, possibly with ancillary data, which can be interpreted as a sequence of observations of one and the same SSO. Note a VSA should be proposed by the Observer, before any attempt to fit an orbit. Typically, the astrometric coordinates are fitted to polynomials of degree either 1 or 2, and the VSA is formed only if the residuals are consistent with the known random component of astrometric error, according to the Error Model. VSA are also called *tracklets*. The Observer should compose and submit VSA in good faith, that is having done the best possible effort to ensure that it can be *true*, that is composed only with true OBS of one and the same SSO. However, some of the VSAs will be *false*, that is containing either true OBS of more than one SSO, or some true and some false OBS, or only false OBS. This is unavoidable and the observers cannot be faulted for submitting some false VSA.

2.3 Computed Quantities

In this subsection we define, although in a mathematically informal way, the quantities and numbers which have to be computed in the algorithms to com-

[4] A trail on a long exposure image may be significant enough to define a DMO. In this case the two ends of the trail should be measured: the problem is, unless there are special provisions (such as asymmetrically interrupted exposures), there is no information on the sense of motion, and two different VSA could be proposed.

plete a discovery and to decide if it is a Discovery according to the definition of Section 3. These are the results of the work of the Orbit Computers.

Definition: Orbit A set of 6 orbital elements and an epoch time, uniquely defining an initial condition of the equations of motion for a Solar System Object. An orbit can be either **Preliminary** if it fits the observations but is by no means determined by them, or **Least Squares** if it is obtained by a fit to the observations with either 5 or 6 free parameters (at most one parameter can be fixed and/or constrained). An orbit also has an estimated absolute magnitude, if some apparent magnitudes are available.

Definition: Identification (ID) A set of VSAs, together with a Least Squares Orbit fitting all of them within the estimated accuracy of the observations. The reason why Identifications are a necessary step is the following. A single VSA almost never allows a Least Squares Orbit to be computed, the rare exceptions occurring in occasion of the discovery of very fast moving, very near objects. Thus multiple VSAs are required to compute Orbits good enough to understand the nature of the object. Given the very limited amount of information available about an object when it has just been detected, a VSA could belong to a large and a priori unknown number of physically distinct objects. To sort out which sets of VSAs belong to the same physical SSO is the task of Identification.

Definition: Observed Arc (ARC) A set of observations, obtained by joining a number of VSA, which can be interpreted as a sequence of DMO of one and the same SSO. It results from an Identification procedure, joining VSAs supplied by one or more Observers.

Definition: Residuals Difference between the Observations and the corresponding predictions, resulting from a given Orbit assumed for the object. A Least Squares Orbit minimizes some *target function* of the Residuals, typically a (weighted) sum of squares. Even a Least Squares Orbit needs to pass some quality control to be acceptable as solution: this implies that the Residuals must pass some statistical test, confirming that they can be interpreted as the effect of observational errors. The simplest such test is based on the value of the standard deviation of the residuals, weighted according to the observations Error Model.

Definition: Error Model A probabilistic description of the astrometric (and photometric) errors, as a function of the observatory, of the instrument used, of the signal to noise of the OBS, of the catalog used for reduction and of any other available ancillary information provided by the Observer. It is typically expressed as a Gaussian Probability Density Function, but in fact the errors do not follow a simple Gaussian distribution, unless a number of outliers is removed; see (Carpino et al. 2003). The Error Model can only be built a posteriori, by statistically analyzing the residuals in the Observed Arcs of SSO with well determined orbits.

Definition: Curvature A measure of the deviation of the Observed Arc from a great circle, traced with uniform speed on the celestial sphere. See Section 6 for a mathematically rigorous definition. The curvature is **Significant** if the deviations of the individual observations from a great circle cannot be due only to observational error, whose statistical properties are assumed from the Error Model. If the Curvature is Significant, in almost all cases a Least Squares Orbit can be computed; otherwise, the Least Squares Orbit either cannot be found or has a too large uncertainty to establish the nature of the object.

Definition: Too Short Arc (TSA) An Observed Arc too short to compute a useful Least Squares Orbit. The minimum of the target function may not exist, may correspond to a very unlikely orbit (e.g., hyperbolic with e significantly larger than 1), may exist but cannot be obtained by differential correction, may have a uncertainty such that it does not allow the nature of the object to be established (see Section 7, in particular Figures 2 and 3). Most VSA are also TSA, because in modern surveys the VSAs are formed with detections separated by a very short time, much less than one day, which implies no Significant Curvature. In some cases an Observed Arc may already be the result of the Identification of ≥ 2 VSA, and still be a TSA: this is often the case for Trans-Neptunian objects.

Definition: Attributable (ATT) A mathematical object describing all the significant information contained in a TSA. For the rigorous definition see Section 6.1, in short an Attributable is a 4-dimensional vector (like an arrow tangent to the celestial sphere) with a date and an optional apparent magnitude. A Least Squares Orbit cannot be computed from a TSA because there are essentially 4 constraints and either 5 or 6 free parameters.

Definition: Arc of Type N An Observed Arc which can be split into exactly N disjoint TSA in such a way that each couple of TSA consecutive in time, if joined, would show a significant curvature. To obtain in all cases a unique value for the Arc Type it is also necessary to specify the method by which the Observed Arcs are to be split. This definition is meant to replace the currently used definition of *N-nighter*, an Observed Arc containing observations belonging to exactly N distinct nights. The new definition has predictive value with respect to the quality of the orbit for all orbital classes of objects, while the old one was useful essentially only for main belt and Trojan asteroids. For main belt asteroids, the definition of Arc of Type N and of N-nighter coincide in most cases; thus the new definition can be considered a generalization of the old one, applicable to a much wider range of orbital parameters from Near Earth Objects (NEO) to Trans-Neptunians. For Trans-Neptunians, 2 nights of observations in most cases form an Arc of Type 1. For NEOs discovered near the Earth a single night of observations often is an Arc of Type ≥ 2 and a Least Squares Orbit with moderate uncertainty can be computed with the first night of data. The (non trivial) algorithm for splitting an Observed Arc into TSAs is discussed in Sections 6.6 and 6.7.

3 The New Definition of Discovery

Definition: Discovery A set of observations of a SSO which are required to form an Observed Arc of Type N with $N \geq 3$. Moreover, there must be a unique least squares orbit (full, that is with 6 free parameters) fitting the data with residuals compatible with the Error Model, and of course the object needs to be a New SSO (see below). It is also required for being a Discovery that the data contain enough photometric information to fit an absolute magnitude. The Observations forming the ARC have to belong to

VSAs which have been submitted to the Data Center, either at once or at different times (by one or more Observers); the Orbit, and the critical Identification (allowing a Type 3 Arc to be built) must either have been submitted to the Data Center by Orbit Computers or have been computed by Data Center itself.

Definition: New SSO A SSO which has not been discovered before, according to the definition above.

Definition: Discovery of a comet A Discovery as above, complemented with enough observational data to prove that there is a directly detectable cometary activity. For example, detecting either a coma, or a tail, or evidence for changes of luminosity with time incompatible with the effect of the rotation of a solid body. It is possible, indeed frequent, that an object is discovered as an asteroid/Centaur and later found to be a comet by observations additional to the astrometry. For a Comet, the photometry requires ancillary data, e.g., specification whether the magnitudes are nuclear.

The above abstract definitions need to be implemented by some procedures, that is, we need to specify who is in charge of checking these conditions. The Arc Type can be computed by using the Observed Arc data only, thus each Observer can check whether an Arc of Type ≥ 3 has been assembled. Large surveys should have the capability of computing Orbits and providing reliable metrics to assess the residuals: in this case they should supply full Identifications, and the Data Center can easily check that the Orbits are reliable and unique. For small observatories this might be to ask too much, in particular having hundreds of observatories computing orbits involves a difficult problem of software standardization. Thus the Observers can submit a Discovery by sending only the Observed Arc and the information that the VSAs it contains belong to the same SSO[5], that is a *proposed identification*, not a complete one; in this case the Data Center should take care of the Orbit computation[6].

4 Discovery credit and Naming Rights

Astronomers like to have their good work officially recognized. For example, discovery credit is a moral reward which can have practical implications in the academic careers of the professional astronomers and is one of the main motivations for the unpaid work of the amateurs. This Section discusses a procedure for assigning discovery credit which aims at being both fair and automatic, without subjective judgment, without dependence from the attention of some official organization, without secret rules and/or proprietary algorithms, and hopefully without leaving ambiguous cases resulting in unpleasant discussions.

Definition: Discovery Credit is assigned to all people and organizations involved in the discovery process, thus in many cases the credit has to be

[5] The Observers submit VSAs with an internal identifying code: they may submit several VSAs with the same code to indicate that these should be identified.

[6] This procedure should not be abused: an Observer cannot send many proposed identification with low reliability, more like guesses, hoping that the Data Center finds among them some good one.

shared. **Priority** may be attributed to some of the contributors, when their contribution has been predominant. The meaning and importance of Priority is similar to being *first author* in a paper published by many authors: it is understood that the results could not have been obtained without the contribution by all authors listed, but some have done work more important than the others.

Discoveries can occur in different ways. A single Observer can submit at once to the Data Center a set of data corresponding to the definition of Discovery as given above. A single Observer can submit the same data over some span of time; several distinct Observers can each contribute part of the data. These data can be, either simultaneously or in a later submission, be assembled into a Discovery level data set by an Orbit Computer. One of the Observers may coincide with the Orbit Computer, and the Orbit Computer may coincide with the Data Center. Discoveries can also be assembled by means of recovery and/or precovery, that is by organized searches based upon ephemerides computed from public data not enough for a Discovery. For example, given an Arc of Type 2 it is possible to predict the position on the sky and/or on archived images with enough accuracy for a targeted search, provided the time span between observations and prediction is not too long. Thus, recovery and precovery is typically joint work by an Orbit Computer and an Observer.

The proposed set of rules for assigning the Discovery Credit is:

1. If a single Observer provides enough data to satisfy the conditions for a Discovery, together with a clear indication that these data are believed to belong to a single SSO, this implies full Discovery Credit, not to be shared.
2. If a single Observer provides enough VSAs to satisfy the conditions for a Discovery, without indication that they belong to the same SSO, then the Discovery Credit is shared with the Orbit Computer submitting Identification and Orbit, but the Observer has Priority.
3. If different Observers have contributed the VSAs necessary for a Discovery, the Discovery Credit is shared among them and among the Orbit Computer(s) performing the Identification(s).
4. The Discovery procedure is closed at the time in which enough data and computations have been submitted. The later contributors do not share the Discovery Credit, although their contribution to the understanding of the object has to be acknowledged.
5. If the observational data forming the Discovery come from different Observers and one of the Observers has supplied data forming an Arc of Type ≥ 2, clearly indicating that these data belong to the same SSO, then he/she has Priority.
6. If the data come from different Observers and all the Observers have supplied Arcs of Type 1, or anyway they have not supplied the proposed identifications, the Orbit Computer providing the full Identification has Priority.
7. If some identifications and/or some predictions for recovery/precovery are performed by the Data Center (acting as Orbit Computer), the Discovery

Credit is assigned also to the Data Center according to the other rules, by using the date of publication in place of the date of submission.

8. Nobody can have Discovery Credit for data and/or computations which were not public at the time the credit has been assigned.

Comment: The purpose of these rules is to reward good practice by the Observers, encouraging them to schedule their observations in such a way that for as many objects as possible Arcs of Type 3 are obtained and identified, possibly by the Observers themselves. When this is not the case, the rules also encourage the work of the Orbit Computers, whose contribution, so far poorly acknowledged, is in some cases essential. The rules encourage the publication of data, even when they are not enough to be a Discovery: with an Arc of Type 2, the Observer may still get Priority, and even with an Arc of Type 1 the Observer being the first to submit can be listed among the discoverers. When data enough for a Discovery have been accumulated, not to publish them immediately means to take the risk of losing all or most of the Discovery Credit[7]. Equally risky is to keep the data secret without analyzing them; this is bad practice, and should be discouraged.

As for the Discovery of a Comet, the required physical observations may be submitted either simultaneously with the astrometry or separately. Thus there are the following additional rules for assigning credit for a Comet Discovery:

9. If an object can be considered Discovered at the time the observations proving its cometary nature are submitted, the Observers (and Orbit Computers) share the Credit for Comet Discovery with the Observer(s) supplying the proof of cometary nature, the latter having Priority.
10. If the Observed Arc of an object is insufficient to form a Discovery at the time the cometary nature is established, the Observer submitting the proof of cometary nature can get Credit (and Priority) for Comet Discovery only if and when the Observed Arc becomes acceptable for Discovery.

Comment: The rules are meant to properly acknowledge the work of the observers performing the critical observations needed to prove the cometary nature. Astrometric observations can never demonstrate the cometary nature of an object, because asteroids and comets can have exactly the same orbit[8]. On the other hand, if an object has a detectable cometary activity, but the orbit could be either main belt or Centaur or nearly parabolic, the nature of the object is not known. It could be a Main Belt Comet (presumably a temporarily activated asteroid), a short periodic comet, a long periodic comet. Thus rule 10 is intended to discourage the bad practice of neglecting astrometric follow up of comets and acknowledge the contribution of the astrometry used to determine the Orbit.

[7] This has the purpose of discouraging *Press Release-driven science*, like when an Observer delays publication with the goal of making a more startling announcement, possibly in some socially convenient venue.

[8] This might mean that some asteroids/Centaurs can be extinct cometary nuclei, but still they are not comets anymore.

The above rules, although they are somewhat more complicated than the old ones because they acknowledge that Discovery is in most cases a collective achievement which cannot be credited to a single actor, do not contradict but just improve upon the astronomical tradition.

Naming Rights are a more complicated case, because the astronomical tradition is somewhat contradictory, with naming rules very different for asteroids/Centaurs and for comets. This generates strange cases when an asteroid/Centaur is later found to be a Comet. For example, comet Wilson-Harrington was identified with asteroid 1979 VA and has a name according to comet rules, implicitly dismissing the credit to the Discoverer of the asteroid (E. Helin) and to the author of the Identification (E.Bowell); (2060) Chiron has a name according to asteroid rules, but it is well known to have cometary activity; (7968) Elst-Pizarro, an asteroid found to have a burst of cometary activity, has an asteroid name coinciding with the names of the discoverers, that is an asteroid name composed with cometary rules. Moreover, the current practice is to give to comets previously discovered as asteroids the name of the survey providing the astrometry, thus contradicting both asteroid naming rules and comet naming rules. Nevertheless, it would not be wise to completely ignore the astronomical tradition, thus there need to be special rules for the naming of comets.

Definition: Naming Right If there is one actor having either full Discovery Credit or Priority, he/she has the right to propose a name for the object discovered. The IAU Small Bodies Naming Committee (SBNC) will generally accept this name, unless it violates some of the IAU naming rules. In exceptional and/or controversial cases the SBNC will decide.

Definition: Naming Right for Comets If and when there is enough evidence that a discovered SSO is a Comet, the comet name is composed by combining the names of all the actors having Discovery Credit, with the Observer having Priority for the Comet Discovery listed first.

5 Comparison with Current Practice

The search for the Solar System Objects has used technologies which have changed substantially with time. In the early times of asteroid discovery, from the times of Piazzi and Olbers to the late 19th century, visual observations were compared with a star map, and this resulted in not more than one observation per night; three observations in three separate nights became then the standard, in order to apply Gauss' method for determining a Preliminary Orbit. In the era of photographic discovery a long exposure plate could reveal a trail, from the center of which a single DMO was obtained, or two plates taken in the same night were “blinked”, thus defining two DMO and a VSA. Thus discoveries were often reported as either one or two observations, and a temporary designation was assigned.

Towards the end of the photographic discovery era there was the bad practice, by some astronomers, of performing the minimum amount of observations required to obtain credit for asteroid discoveries: one night of observations with only two exposures per field was enough to have a number of discoveries credited and published in the Minor Planets Circulars, a publication with IAU sponsorship. As a result in 1991 Brian Marsden, director of the MPC, decided not to publish the data on the objects observed only during a single night on the Minor Planet Circulars, thus denying officially recorded credit to the discoverer. This decision was justified at the time it was taken. Over a longer time span its positive effects were lost: now in the MPC observation data files there are many more 2-nighters than 3-nighters, maybe because again some Observers have optimized their searches for discovery credit.

However, within a very short time the technology used by both Observers and Orbit Computers changed radically. The Orbit Computers wanted computer readable files, to be transfered through Internet, rather than paper Circulars, to be consulted by eye. The Observers started using the CCD technology and very soon the use of photographic plates for discovery declined, even the historic collections of plates needed to be digitized to be used as digital observations (e.g., for precovery). As an example, the previously unknown population of Trans Neptunian Objects (TNO) was discovered only by deep CCD images and software blinking (typically with images from different nights).

In 1998 a new generation of asteroid/comet surveys become operational and produced automatically an enormous number of VSAs. In 1999 the MPC restarted to disseminate computer readable files of observations, but the 1-nighters were excluded. The MPC considered it was their task to identify at least two 1-nighters to form an arc which would be accepted as a discovery. The instructions given by the MPC to the Observers did not require identifications, they actually discouraged the Observers from trying to identify their own VSAs.

It was immediately obvious that this setup would result in the loss of too many newly discovered objects, in particular the most interesting ones, like the Near Earth Objects (NEO). The big surveys were optimized to cover large portions of the sky down to a very dim limiting magnitude, but would become inefficient if used for targeted follow up. Thus the Near Earth Objects Confirmation Page (NEOCP) was setup by the MPC. On the basis of some VSA with large proper motion, most likely to belong to a NEO, ephemerides were computed (typically by using 4-parameter orbits, as in the method by Väisälä) and a request for follow-up observations was posted on the Web. This was effective in stimulating the collaboration of observatories with less telescope power than the major surveys, including many amateurs who had already completed the transition to fully digital astrometry.

It has to be acknowledged that this improved setup, including the NEOCP, has worked well in most cases, and indeed the rate of discovery went up considerably, not only for MBA (and Trojans) but also for NEO. However, there were four major shortcomings. First, the contributions from Orbit Computers outside the MPC was not requested, and not always acknowledged even when submitted. Second, the performance of the NEOCP depended in a critical way upon the skill of the MPC personnel in selecting the “interesting” objects and in predicting their recovery positions. Third, the rules established by the MPC on discovery credit were violated by the MPC itself. Fourth, the discovery rules used by the MPC were totally inadequate for the case of TNO.

Let us discuss these four points. After 1999, with public access to the files of past observations, some groups of independent Orbit Computers started competing with the MPC in the search for identifications, and indeed they were able to find a large number (tens of thousands) of additional identifications, based on the data already scanned for this purpose by the MPC[9]. In this way they gave a practical verification of the general principle that the lack of open competition in research results in slower scientific progress.

As for the second point, the observations used to compute the ephemerides posted on the NEOCP were typically 1-nighters, and anyway the observations were not made public. It does not matter how skilled are the people holding a monopoly on the data, they will never be able to perform as well as the entire scientific community struggling to achieve the best results. Indeed, sometimes the MPC failed either to select an "interesting" object or to predict the ephemerides accurately enough for recovery. This did not happen very often, but still there were some embarrassing cases, such as the one of 2000 SG_{344}, reported three times as a VSA from two different observatories and three times lost (twice after posting on the NEOCP), only to be rediscovered several months later. This case was noticed and raised some heated discussion because this asteroid had at the time the possibility of impacting the Earth, as reported by the NEODyS/CLOMON impact monitoring system[10].

The third point is especially relevant for the present paper. If a request for follow up is posted for an object observed only in one night, that is an object without a designation, then who gets the discovery credit? The MPC started to assign discovery credit to the Observer providing the VSA which was selected by the MPC itself for NEOCP posting, even if the same observatory did not contribute a second night of observations. This violates the 1991 rule against credit to 1-nighters, which was never formally abolished and is still applied to MBA. Moreover, a previous VSA, supplied by some other observatory and belonging to the same object, might have been ignored. After all, a NEO is far from the Earth before coming close, thus the same object may well have been detected while moving slowly before being observed while moving fast. Thus the assignment of discovery credit depended from discretionary choices of the MPC, for which never there were documented algorithms: they were performed based on the experience of the MPC personnel. In conclusion, the assignment of discovery credit precisely for the highest priority discoveries, the ones of NEOs, was performed against the rules and in a way which could in some cases be unfair, resulting in several unpleasant controversies. This is a serious problem, since the number of NEO discoveries has been the main metric used by some funding agencies to assess the comparative performance of the surveys.

[9] The two most successful groups in proposing new identifications were Doppler et al. and Sansaturio et al., but others contributed as well.
[10] It still has a possibility of impacting the Earth in 2070–2071, but its size is small, thus there is no cause for great concern; see http://newton.dm.unipi.it/neodys/

The fourth point is also very relevant for this paper. To discover an object with very slow proper motion, such as a TNO (also for a Centaur, and in some cases for a long period comet), observations over several nights are required anyway. The availability of two nights has no special meaning, actually in many cases the interval of one full day (or more) is used for blinking, that is two (or more) nights are the time span for a VSA. Even such a 2-night VSA in most cases does not allow an orbit with more than 4 parameters, i.e, it is a TSA. Nevertheless, the Attributable can be used to compute a prediction (by extrapolating along the great circle) good enough to recover the object several days later, in some cases even a few weeks later. Thus the discovery credit can be assigned, by the current rules, before any meaningful orbit is available. After such an announcement other observers can easily follow up with astrometry, determining an Orbit, and with other observations, such as searches for satellites, spectra and size determinations. These follow up observers may get more scientific recognition, while the contribution of the officially credited discoverer might be forgotten by all[11]. Given the time scales needed to accumulate the most critical information on TNO, that could be years, anyway the publication dilemma facing the Observers cannot be entirely solved. Therefore we would like, with our proposal, to at least ensure that the discovery credit is assigned only when an Orbit good enough to discriminate the different populations (classical TNO, Plutinos, scattered disk, Centaurs, etc.) is available, and to reward the patient work needed to collect the minimum set of astrometry.

6 Mathematical Specifications

6.1 Attributables

When a celestial body is observed, let $(\rho, \alpha, \delta) \in \mathbb{R}^+ \times [-\pi, \pi) \times (-\pi/2, \pi/2)$ be spherical coordinates for the topocentric position. The angular coordinates (α, δ) are defined by a reference system selected in an arbitrary way. In practice we use for α the right ascension and for δ the declination with respect to an equatorial reference system (e.g., J2000). We shall call *attributable* a vector $A = (\alpha, \delta, \dot{\alpha}, \dot{\delta}) \in [-\pi, \pi) \times (-\pi/2, \pi/2) \times \mathbb{R}^2$, representing the topocentric angular position and velocity of the body at a time $\bar{t}$. Optionally an average apparent magnitude may be available.

The procedure to compute an attributable, if there are $m \geq 3$ observations (at different times), is as follows. Given the observed values $(t_i, \alpha_i, \delta_i)$ for $i = 1, m$ we can fit both angular coordinates as a function of time with a polynomial model: in the cases of interest a degree 2 model is satisfactory

$$\alpha(t) = \alpha(\bar{t}) + \dot{\alpha}(\bar{t})\ (t - \bar{t}) + \frac{1}{2}\ddot{\alpha}(\bar{t})\ (t - \bar{t})^2$$
$$\delta(t) = \delta(\bar{t}) + \dot{\delta}(\bar{t})\ (t - \bar{t}) + \frac{1}{2}\ddot{\delta}(\bar{t})\ (t - \bar{t})^2$$

[11] Even by the MPC, as in some recent cases.

with $\bar{t}$ the mean of the t_i; the solution $(\alpha, \dot\alpha, \ddot\alpha, \delta, \dot\delta, \ddot\delta)$ is obtained with the standard formulae of the least squares problem, together with the two 3×3 covariance matrices $\Gamma_\alpha, \Gamma_\delta$. Note that the observations can be weighted[12].

The second time derivatives are computed as an insurance against the possibility that a linear fit is not a good representation of the short arc data, but the attributable contains only the averages and rates of angular motion[13]. The marginal covariance matrix of A, whatever the values of $(\ddot\alpha, \ddot\delta)$, is obtained by extracting the relevant 4×4 sub-matrix:

$$\Gamma_A = [\gamma_{ik}]_{i,k=1,4}$$
$$\gamma_{1,1} = \gamma_{\alpha,\alpha} \quad \gamma_{2,2} = \gamma_{\delta,\delta} \quad \gamma_{3,3} = \gamma_{\dot\alpha,\dot\alpha} \quad \gamma_{4,4} = \gamma_{\dot\delta,\dot\delta}$$
$$\gamma_{1,3} = \gamma_{3,1} = \gamma_{\alpha,\dot\alpha} \quad \gamma_{2,4} = \gamma_{4,2} = \gamma_{\delta,\dot\delta}$$

with the other coefficients 0; the normal matrix is $C_A = \Gamma_A^{-1}$. The matrices C_A, Γ_A defined in this way are positive definite.

6.2 Unique names

If a VSA, or tracklet, is the unit of observational data to be submitted for publication, it must have a unique name. It would be helpful to have an algorithm to compute this unique name in such a way that each Observer can assign it, without the need of complicated data negotiations with the Data Center. The unique name could be a string encoding the observatory code, the date of the observations, and some function of the observed coordinates, in such a way that duplicate names either do not occur or are rare to the point of not being cause of concern, and being handled by a simple recovery procedure.

One such algorithm has been defined and programmed by O. Arratia and M.E. Sansaturio, and it is adequate for the current data volume. It uses base 64 encoding in printable ASCII characters, with two positions for the observatory code, three for the MJD date, four to encode the arc seconds of α and δ (after subtracting an integer number of degrees). In this way the name length is not more than the 9 characters of the current IAU temporary designations. This software is available as part of the OrbFit free software system (see Section 8).

With the next generation surveys, the 9 character encoding as above will not be enough, because of the expected 100-fold increase of the data rate. A generalization of the Arratia-Sansaturio algorithm with 11 characters should be enough for the next generation of surveys. To avoid problems with false VSAs, the values of α, δ to be encoded should be the ones of the attributables, and the encoding may contain also some information on $\dot\alpha, \dot\delta$.

[12] We are assuming that the α and δ error component of an astrometric observation are not correlated, otherwise the 6×6 covariance matrix of all the variables could be full. This assumption would fail if the timing was a significant source of error.

[13] If there are only $m = 2$ observation a simple linear fit has to be used.

When the VSAs are assembled into Observed Arcs, it is important that the unique names are preserved. Even when an asteroid is numbered, the *paper trail* of the successive identifications leading to the current Observed Arc and Orbit needs to be stored, and this is impossible if the VSA unique name has been erased. For example, when a false Identification is later detected on the basis of new data, reference to the removed VSA must be again by its unique name.

6.3 Geodetic curvature and acceleration

The heliocentric position of the SSO is the vector $\mathbf{r} \in \mathbb{R}^3$ and the topocentric position is

$$\boldsymbol{\rho} = \rho\, \hat{\boldsymbol{\rho}} = \mathbf{r} - \mathbf{q}$$

where $\mathbf{q}$ is the heliocentric position of the observer, $\hat{\boldsymbol{\rho}}$ is the unit vector defining the observation direction, ρ the distance.

Following (Danby 1985), let us define an orthonormal basis adapted to the path on the celestial sphere (on which $\hat{\boldsymbol{\rho}}$ lies)

$$\mathbf{v} = \frac{d\hat{\boldsymbol{\rho}}}{dt} = \eta\, \hat{\mathbf{v}} \quad , \quad \hat{\mathbf{v}} \cdot \hat{\boldsymbol{\rho}} = 0$$

where $\eta = \|\mathbf{v}\|$ is the *proper motion*. Note that, by using the *arc length parameter* s, defined by $ds/dt = \eta$, we have $d\hat{\boldsymbol{\rho}}/ds = \hat{\mathbf{v}}$ and the derivative with respect to the arc length, which we indicate with a prime $d\hat{\mathbf{v}}/ds = \hat{\mathbf{v}}'$, has the properties

$$\hat{\mathbf{v}}' \cdot \hat{\mathbf{v}} = \frac{1}{2}\, \frac{d}{ds} \|\hat{\mathbf{v}}\|^2 = 0$$
$$\hat{\mathbf{v}}' \cdot \hat{\boldsymbol{\rho}} = \frac{d}{ds}\left[\hat{\mathbf{v}} \cdot \hat{\boldsymbol{\rho}}\right] - \hat{\mathbf{v}} \cdot \hat{\boldsymbol{\rho}}' = -1$$

We can use the third orthogonal vector $\hat{\mathbf{n}} = \hat{\boldsymbol{\rho}} \times \hat{\mathbf{v}}$ to express $\hat{\mathbf{v}}'$ as

$$\hat{\mathbf{v}}' = -\hat{\boldsymbol{\rho}} + \kappa\, \hat{\mathbf{n}}$$

where the quantity κ is the *geodetic curvature* of the path. It measures the deviation of the path from a great circle (a geodetic on the sphere).

Another component of the second time derivative of the path $\hat{\boldsymbol{\rho}}(t)$ on the sphere is the along track *acceleration*, that is

$$\frac{d^2 \hat{\boldsymbol{\rho}}}{dt^2} \cdot \hat{\mathbf{v}} = \frac{d}{dt}\left(\eta\, \hat{\mathbf{v}}\right) \cdot \hat{\mathbf{v}} = \left(\dot{\eta}\, \hat{\mathbf{v}} + \eta^2\, \hat{\mathbf{v}}'\right) \cdot \hat{\mathbf{v}} = \dot{\eta}\ .$$

The third component of curvature is simply the curvature of the sphere, that is it corresponds to the formula $\hat{\mathbf{v}}' \cdot \hat{\boldsymbol{\rho}} = -1$.

6.4 Computation of curvature

To compute geodetic curvature and acceleration, starting from the values of $(\alpha.\delta, \dot{\alpha}, \dot{\delta}, \ddot{\alpha}, \ddot{\delta})$ obtained by polynomial fitting of the observations, we use the orthogonal frame $\{\hat{\boldsymbol{\rho}},\ \hat{\boldsymbol{\rho}}_\alpha,\ \hat{\boldsymbol{\rho}}_\delta\}$

$$\begin{aligned}\hat{\boldsymbol{\rho}} &= (\cos\delta\cos\alpha, \cos\delta\sin\alpha, \sin\delta)\\ \hat{\boldsymbol{\rho}}_\alpha &= \frac{\partial\hat{\boldsymbol{\rho}}}{\partial\alpha} = (-\cos\delta\sin\alpha, \cos\delta\cos\alpha, 0)\\ \hat{\boldsymbol{\rho}}_\delta &= \frac{\partial\hat{\boldsymbol{\rho}}}{\partial\delta} = (-\sin\delta\cos\alpha, -\sin\delta\sin\alpha, \cos\delta)\ .\end{aligned}$$

This is not an orthonormal frame, since the length of the vectors is[14]

$$\|\hat{\boldsymbol{\rho}}\| = \|\hat{\boldsymbol{\rho}}_\delta\| = 1 \quad , \quad \|\hat{\boldsymbol{\rho}}_\alpha\| = \cos\delta\ .$$

In this frame

$$\begin{aligned}\hat{\mathbf{v}} &= \hat{\boldsymbol{\rho}}' = \alpha'\ \hat{\boldsymbol{\rho}}_\alpha + \delta'\ \hat{\boldsymbol{\rho}}_\delta\\ \hat{\mathbf{n}} &= \hat{\boldsymbol{\rho}} \times (\alpha'\ \hat{\boldsymbol{\rho}}_\alpha + \delta'\ \hat{\boldsymbol{\rho}}_\delta) = -\frac{\delta'}{\cos\delta}\ \hat{\boldsymbol{\rho}}_\alpha + \alpha'\cos\delta\ \hat{\boldsymbol{\rho}}_\delta\\ \hat{\mathbf{v}}' &= (\alpha''\hat{\boldsymbol{\rho}}_\alpha + \delta''\hat{\boldsymbol{\rho}}_\delta) + \left(\alpha'^2\hat{\boldsymbol{\rho}}_{\alpha\alpha} + 2\alpha'\delta'\hat{\boldsymbol{\rho}}_{\alpha\delta} + \delta'^2\hat{\boldsymbol{\rho}}_{\delta\delta}\right)\end{aligned}$$

where the double prime indicate the second derivative with respect to the arc length, and the second derivative vectors are

$$\begin{aligned}\hat{\boldsymbol{\rho}}_{\alpha\alpha} &= \frac{\partial^2\hat{\boldsymbol{\rho}}}{\partial\alpha^2} = (-\cos\delta\cos\alpha, -\cos\delta\sin\alpha, 0)\\ \hat{\boldsymbol{\rho}}_{\alpha\delta} &= \frac{\partial^2\hat{\boldsymbol{\rho}}}{\partial\alpha\,\partial\delta} = (\sin\delta\sin\alpha, -\sin\delta\cos\alpha, 0)\\ \hat{\boldsymbol{\rho}}_{\delta\delta} &= \frac{\partial^2\hat{\boldsymbol{\rho}}}{\partial\delta^2} = (-\cos\delta\cos\alpha, -\cos\delta\sin\alpha, -\sin\delta)\end{aligned}$$

To compute the geodetic curvature we need the scalar products[15]

$$\begin{array}{ll}\hat{\boldsymbol{\rho}}_{\alpha\alpha}\cdot\hat{\boldsymbol{\rho}}_\alpha = 0 = \Gamma_{\alpha\alpha,\alpha} & \hat{\boldsymbol{\rho}}_{\alpha\alpha}\cdot\hat{\boldsymbol{\rho}}_\delta = \sin\delta\cos\delta = \Gamma_{\alpha\alpha,\delta}\\ \hat{\boldsymbol{\rho}}_{\alpha\delta}\cdot\hat{\boldsymbol{\rho}}_\alpha = -\sin\delta\cos\delta = \Gamma_{\alpha\delta,\alpha} & \hat{\boldsymbol{\rho}}_{\alpha\delta}\cdot\hat{\boldsymbol{\rho}}_\delta = 0 = \Gamma_{\alpha\delta,\delta}\\ \hat{\boldsymbol{\rho}}_{\delta\delta}\cdot\hat{\boldsymbol{\rho}}_\alpha = 0 = \Gamma_{\delta\delta,\alpha} & \hat{\boldsymbol{\rho}}_{\delta\delta}\cdot\hat{\boldsymbol{\rho}}_\delta = 0 = \Gamma_{\delta\delta,\delta}\ .\end{array}$$

Then the geodetic curvature is

$$\kappa = \hat{\mathbf{v}}'\cdot\hat{\mathbf{n}} = (\delta''\alpha' - \alpha''\delta')\cos\delta + \alpha'\ (1+\delta'^2)\sin\delta$$

as a function of the derivatives with respect to the arc length; to convert into an expression containing the time derivatives we need to use

$$\alpha'' = \frac{1}{\eta}\ \frac{d}{dt}\ \left(\frac{\dot{\alpha}}{\eta}\right) = \frac{\eta\ \ddot{\alpha} - \dot{\eta}\ \dot{\alpha}}{\eta^3}\ ,$$

[14] That is, the Riemannian metric of the sphere is $ds^2 = \cos^2\delta\ d\alpha^2 + d\delta^2$.
[15] That is, the Riemannian connection of the sphere, as expressed by the Christoffel symbols $\Gamma_{ij,k}$.

where η is computed as $\sqrt{\dot\alpha^2\ \cos^2\delta+\dot\delta^2}$, and the analog for δ''. The terms containing $\dot\eta$ cancel out, giving

$$\kappa=\frac{1}{\eta^3}\left[(\ddot\delta\dot\alpha-\ddot\alpha\dot\delta)\cos\delta+\dot\alpha(\eta^2+\dot\delta^2)\sin\delta\right]\ .$$

To compute the acceleration we use the second derivative of the path

$$\frac{d^2\hat{\boldsymbol\rho}}{dt^2}=\left(\ddot\alpha\ \hat{\boldsymbol\rho}_\alpha+\ddot\delta\ \hat{\boldsymbol\rho}_\delta\right)+\left(\dot\alpha^2\ \hat{\boldsymbol\rho}_{\alpha\alpha}+2\ \dot\alpha\ \dot\delta\ \hat{\boldsymbol\rho}_{\alpha\delta}+\dot\delta^2\ \hat{\boldsymbol\rho}_{\delta\delta}\right)\ ,$$

then compute the along track component

$$\dot\eta=\frac{d^2\hat{\boldsymbol\rho}}{dt^2}\cdot\hat{\mathbf v}=\frac{\ddot\alpha\ \dot\alpha\ \cos^2\delta+\ddot\delta\ \dot\delta-\dot\alpha^2\ \dot\delta\ \cos\delta\ \sin\delta}{\eta}\ .$$

Given these formulas, it is possible to compute the covariance matrix of the quantities $(\kappa,\dot\eta)$ by propagation of the covariance matrix of the angles and their derivatives with the matrix of partial derivatives computed from the above formulae for κ and $\dot\eta$

$$\Gamma_{\kappa,\dot\eta}=\frac{\partial(\kappa,\dot\eta)}{\partial(\alpha,\dot\alpha,\ddot\alpha,\delta,\dot\delta,\ddot\delta)}\ \begin{bmatrix}\Gamma_\alpha & \underline{0}\\ \underline{0} & \Gamma_\delta\end{bmatrix}\ \left[\frac{\partial(\kappa,\dot\eta)}{\partial(\alpha,\dot\alpha,\ddot\alpha,\delta,\dot\delta,\ddot\delta)}\right]^T\ .$$

This matrix has been computed with a program generated by the algebraic manipulator *Maple V*; its derivation will be fully documented elsewhere. From the covariance matrix we can assess the significance of the estimated values for the curvature: if the standard deviation of κ, computed from the covariance matrix as $\sqrt{\gamma_{\kappa\kappa}}$, is larger than $|\kappa|$ the true curvature could well have a sign opposite to the the one of the estimated value.

As it is known from the classical theory of preliminary orbit determination (Danby 1985), (Plummer 1918), the geodetic curvature and the acceleration are related to the range and range rate

$$\kappa=\kappa(\rho,A)\qquad,\qquad\dot\eta=\dot\eta(\rho,\dot\rho,A)\tag{1}$$

but the map $\rho\mapsto\kappa$ (for a given attributable A) is not always invertible, because ρ is the solution of a degree 8 equation which can have multiple real roots. Some of these roots are *admissible*, in the sense of corresponding to possible positions of the observed object. In particular near quadrature (with elongations less than 116°.5) in the majority of cases there are two admissible distances ρ. For each value of ρ a unique value of $\dot\rho$ can be computed. The set of 6 coordinates $(A,\rho,\dot\rho)$ uniquely defines an initial condition, thus a preliminary orbit.

When the observations directions are near the opposition, there is only one admissible distance ρ and one preliminary orbit. Using this preliminary orbit as first guess in the differential correction procedure a least squares orbit can be found, with low residuals if the arc is short, because it contains little information beyond the second derivatives of the angles.

When the observations are near quadrature, there are often two preliminary orbits, and starting from each of the two a least squares orbit can be found by differential correction. If the arc is short both least squares solutions can have good residuals, thus in many cases it is not possible to discriminate between the two. These two orbits are very different, such as one Main Belt and one Aten (Boattini et al. 2006); the difference is well beyond what would appear from the covariance matrix of both solutions. The target function has in this cases two local minima; an example is given in Section 6 of (Milani et al. 2005a).

This is the motivation for the provision, contained in the definition of Discovery of this paper, for checking not only the existence and the quality, but also the uniqueness of the least squares solution. The existence of an alternative Least Squares solution can be checked as in (Boattini et al. 2006) by exploiting the algebraic properties of Equation (1). In practice, this implies that for Discovery some additional follow up will be needed, on top of a Type 3 Arc, for a number of cases observed near the quadratures.

The discussion above applies when the geodetic curvature is large, or anyway much larger than its uncertainty. On the contrary, if the standard deviation $\sqrt{\gamma_{\kappa\kappa}}$ is larger than the estimated value of κ, the range ρ is essentially undetermined (indeed $\kappa \to 0$ for $\rho \to +\infty$). In such case the Observed Arc does not provide information on $(\rho, \dot{\rho})$, but only contains the information described by the attributable A.

6.5 Error Model

The definition of the covariance matrices, thus also the definition of significant curvature in the previous Section, depend upon assumptions on the distribution of astrometric errors for each observing instrument, also as a function of the observing conditions and of the astrometric data processing. This model cannot be assumed a priori, but must be the result of careful statistical tests on the residuals for well determined orbits (e.g., for numbered asteroids); the method has been established and tested in (Carpino et al. 2003). Indeed, we are using for the tests of Section 7 the error model from this paper. However, this is just the first approximation of a future procedure to establish a reliable observation error model, because the work so far has been limited by the lack of essential information, not contained in the public MPC data.

Statistical analysis needs to be performed on homogeneous data sets, which can be presumed to have one and the same Probability Density Function (PDF) of the error distribution. This is essential, as it is clear from a simple example: the combination of two different Gaussian PDF is not a Gaussian. Three pieces of information are essential to bin the data of a given observatory in homogenous subsets: which instrument was used (for the observatories with multiple instruments), the signal to noise ratio of each individual observation, and the star catalog used for astrometric reduction. Of these three pieces of information, the first one is sometimes available in the data made public by the MPC, the other two are not available.

At the IAU General Assembly, held in Manchester in 2000, there was a public discussion on the need of a new format for the submission to the MPC and for dissemination from the MPC of the observational data. The discussion between the Orbit Computers and the MPC ended in an agreement on the new format, whose specifications have been published. However, the new standard has yet to be implemented, and of course the transition phase is going to be long and difficult for the MPC, for the Observers and for the Orbit Computers alike. Waiting for this, we have to use what we have.

The error model is essential because it provides the correct weighting for the individual observations. Moreover, as established by (Carpino et al. 2003), the data also contain outliers, bias and correlations. Bias can be removed by subtracting, correlations can be accounted by replacing the simple sum of squares of the residuals with a non-diagonal quadratic form. There is the need of a formal procedure to discard some observations considered outliers, also outlined in the same paper. In the definition of significant curvature we are not considering the outliers (identified in a previous iteration of orbit determination). Moreover, the MPC does mark some observations as unreliable (either by giving few significant digits, or by a special code) and these are not considered.

The procedure to build and maintain a reliable and up to date Error Model should be one of the priority tasks of the IAU Data Center, nevertheless it will not be immediately available. Thus it is unavoidable to begin by applying the new Discovery definition based upon a simplistic Error Model (like one arc second weight for all current observations). This is somewhat unfair to the Observers doing the effort to produce high quality astrometry, thus needs to be a temporary fix, to be replaced as soon as possible by a real model. In the future, the Error Model shall be regularly updated to account for the new Observers and for the improvements in accuracy.

6.6 Arc Type

It is not always easy to establish whether a given Observed Arc can be used to compute a least squares orbit. The procedure needed involves the solution of a nonlinear least squares, which in practice can only be solved by an iterative procedure, starting from some preliminary orbit. The classical method is *differential corrections*, a variant of Newton's method, but other methods are available. In particular, *constrained differential corrections* (Milani et al. 2005a) are an effective improvement upon the classical procedure, allowing a least squares solution with 5 free parameters; this is especially effective for obtaining orbits from a Type 2 Arc. Other more reliable, but also more computationally intensive methods are known and could be used (Sansaturio et al. 1996). In practice, the methods for orbit determination are computationally expensive and the result may depend, in a very unstable way, upon the initial preliminary orbit and upon the details of the method used; thus, not having found a least squares solution does not prove it does not exist. For all these reasons, we prefer to use an approximate definition of Too Short Arc which can be explicitly computed at negligible computational cost and with not too many details to be specified.

We use as an approximate criterion for deciding that an Observed Arc is a TSA the significance of its curvature. Given a control value χ^2_{min}, we compute the χ^2 for the geodetic curvature and acceleration

$$\chi^2 = \begin{bmatrix} \kappa \\ \dot{\eta} \end{bmatrix}^T C_{\kappa,\dot{\eta}} \begin{bmatrix} \kappa \\ \dot{\eta} \end{bmatrix} \quad , \quad C_{\kappa,\dot{\eta}} = \Gamma^{-1}_{\kappa,\dot{\eta}} \ .$$

Definition: Significant curvature is a property of an Observed Arc if, after computing a single attributable for all the observations of the arc, from the equation above we get a value such that $\chi^2 > \chi^2_{min}$. The choice of the value of χ^2_{min} is not obvious, in the tests of Section 7 we have experimented with different values. Then the approximate definition of TSA can be as follows: an Observed Arc is a TSA when it does not have significant curvature.

Note that the definition of significant curvature means that the origin is outside the confidence region (with confidence parameter χ^2_{min}) of the vector $[\kappa, \dot{\eta}]$; the two scalar quantities κ and $\dot{\eta}$ could have marginal uncertainties larger than their absolute value, but the two cannot be simultaneously zero. In this approach the same relevance is given to κ and to $\dot{\eta}$, although Equation (1) does indicate that κ is more relevant for the computation of a preliminary orbit. Our tests, described in Section 7, suggest that also for the possibility of computing a least squares orbit κ is more relevant than $\dot{\eta}$, but the difference is not critical.

The main problems with a definition of Arc Type based on this approximate definition of TSA are two: the choice of the control value χ^2_{min}, and the fact that some Observed Arcs may contain enough information for a significant third derivative of the angles with respect to time. If the latter is the case, the residuals of a quadratic fit have a characteristic Z shape (see Figure 1), which is significant if the standard deviation of the (normalized) residuals is larger than some RMS_{min}.

Thus there is a choice to be made in the definition of TSA. Let us suppose an Observed Arc has no significant curvature but has significant Z shape. If the definition of TSA is based only upon curvature, then this arc would be a TSA. However, this arc contains much more information than the Attributable, and indeed in many cases a Least Squares Orbit can be computed: the example of Figure 1 has a marginally significant κ, but the distance ρ can be accurately determined (with some method different from Laplace's)[16]. Therefore we have chosen to adopt the following formal definition of TSA, replacing the informal one of Section 2.3.

Definition: Too Short Arc (TSA) An Observed Arc such that there is no significant curvature: $\chi^2 < \chi^2_{min}$ and no significant Z shape, that is the RMS of a fit to a quadratic function of time is $RMS < RMS_{min}$.

An additional technicality is that there needs to be a minimum size (in time and in angular distance on the celestial sphere) below which an Observed Arc cannot be split, to avoid that especially noisy data (with astrometric errors larger than expected from the current Error Model) result in a spurious increase in the Arc Type[17].

[16] In the test of Section 7.1 the cases for which the split was based on Z shape only were 7.4% of the total.

[17] For the tests of Section 7 we are using a minimum time span of 30 minutes and a minimum arc length of 1 arc minute.

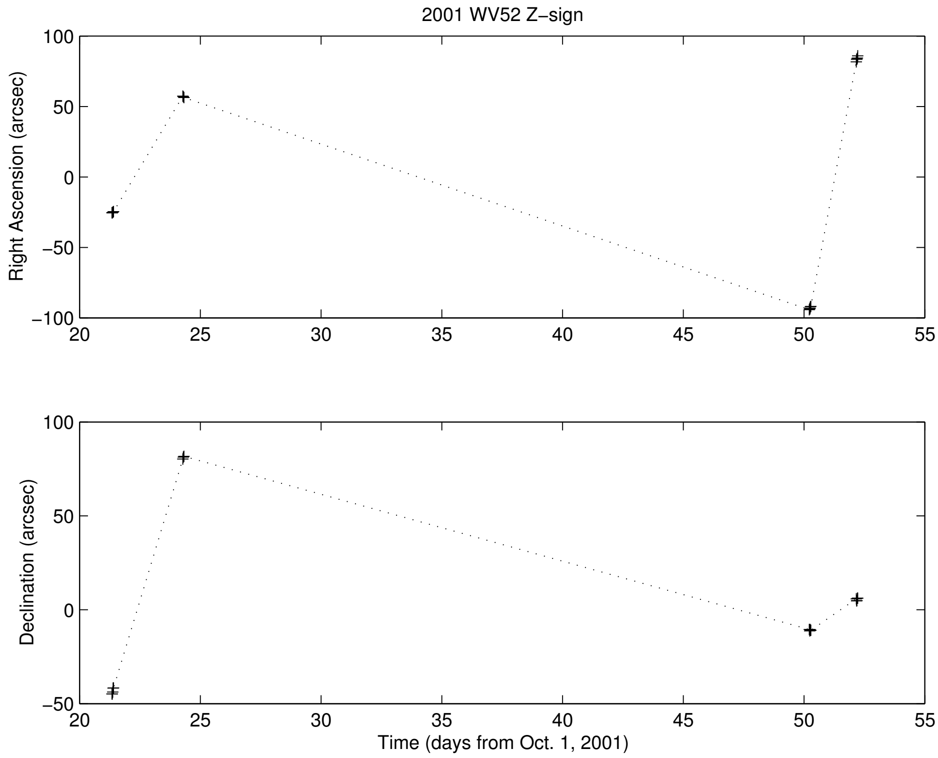

Fig. 1 An example of the "Z shape", for the asteroid 2001 WV_{52}, with 4 nights of observations forming a Type 2 Arc; the Main Belt orbit is well determined. The plots show the residuals after removing only the linear trends: the effect of the third time derivative is apparent because the arc time span is quite long. The value of κ is only marginally significant, but $\dot{\eta}$ is very significant, as can be appreciated from the different slopes between the first and the third segment in the Z.

6.7 How to compute the Arc Type

One thing is an abstract definition, and another is to be able to compute explicitly the Arc Type of a given Observed Arc. A *recursive procedure* to compute the Arc Type could be as follows: if the arc has significant curvature and/or a significant Z shape, it is split into two arcs by selecting the largest time gap between two observations. If the two sub-arcs have no significant curvature and no significant Z shape, the procedure ends and the Arc Type is 2. Otherwise, the same procedure is applied to the two sub-arcs, and the Arc Type is the sum of the Arc Types of the sub-arcs. The recursion must terminate because the number of observations in the sub-arcs decreases, and a sub-arc with < 3 observations can have neither curvature nor Z shape.

The above procedure has the advantage of being unambiguously defined and of giving always a unique integer Arc Type. However, it is not always true that splitting at the largest time gap gives the decomposition in the minimum number of TSA: it is possible to manufacture a counterexample, which could be realistic in some special cases, when there is a retrograde loop. The procedure can be improved by checking, after the arc has been decomposed by the recursive procedure above, whether two consecutive sub-arcs can be joined in a single TSA, with neither significant curvature nor significant Z shape; if this condition occurs, the two sub-arcs are joined and the Arc Type is decreased by one. We are using in our tests the splitting by largest gap, but with this additional step[18].

In conclusion, we have a definition of Arc Type which is rigorous, operative and easy to compute. It is significant for assessing the difficulties which will be found in orbit determination, but we cannot expect that it predicts in a fail proof way the cases in which a good orbit can be computed. The experiments of Section 7 will be important in assessing how serious are these limitations.

7 Numerical Experiments

We have performed a number of tests of the new definition of Discovery, in particular to assess how correlated it is with the uncertainty of the orbit which can be determined. In two large scale tests we have used the astrometric data for unnumbered asteroids made public by the MPC in November 2005 and again in March 2006. We have, however, run the two tests with slightly different parameters, to test the sensitivity to the values of χ^2_{min} and RMS_{min}.

7.1 Test with weak definition of Arc Type

The dataset released in November 2005 contained about 8.5 million observations assigned to $280,624$ Observed Arcs; however, we had to discard $5,612$ designations with only one observation, and $3,419$ with only two observations, which obviously are not Discoveries.

We have first computed the Arc Types, following the definition in Section 6.6, for all Observed Arcs with an arc time span not exceeding 180 days, that is $168,754$ Arcs with $2,462,749$ Observations. We have used the values $\chi^2_{min} = 1$ and $RMS_{min} = 3$ for the controls, that is a *weak* definition of Arc Type, in which either a curvature barely above the noise or a moderate Z shape are enough to split an arc.

The results of the test are summarized in Table 1. In the columns of the Table we give the count, for each Arc Type, of the Observed Arcs for which the curvature is "good", that is $\chi^2 > 100$ (larger than the χ^2_{min} required for splitting), and for which we have been able to compute a "good orbit", that is a least squares orbit with standard deviations < 0.1 for the perihelion q (in AU), < 0.1 for the eccentricity e and $< 10°$ for the inclination I. The values

[18] It is possible to give alternative definitions of Arc Type by using different splitting rules and different additional control steps. However, the definition needs to be kept simple, and the utmost care must be taken to maintain the essential property that the Arc Type is uniquely defined by the set of Observations.

used to define "good" are quite arbitrary, but meaningful: we have chosen an uncertainty in q of 0.1 AU that allows Main Belt Asteroids (MBA) to be discriminated from Near Earth Objects (NEO), and also MBA from Trojans.

Table 1 The performance of the Observed Arc Types in orbit computation, test with weak definition of Arc Type and November 2005 data.

Arc Type	Good orbit Good curv.	Good orbit Poor curv.	Bad/no orbit Poor curv.	Bad/no orbit Good curv.	Total
1	0	12	17,675	11	17,698
2	3,927	232	61,987	6,460	72,606
3	14,961	127	2,853	4,542	22,483
4	13,076	46	304	784	14,210
5	10,289	14	25	110	10,438
6	7,885	3	2	20	7,910
> 6	23,398	1	0	10	23,409

Some conclusions are quite clear from Table 1. The Arcs of Type 1 are almost never enough to compute useful orbits, although in fact we have been able to compute a full least squares orbit for 52.8% of them (a constrained 5 parameter orbit for another 17.9%). These orbits are undetermined to the extreme (as shown in Figure 2, top left), to the point that most of them do not give any clue on the nature of the object: they have to be considered as extrapolation tools, useful only for short term recovery/precovery, slightly better than just linear extrapolation (along the great circle defined by the Attributable). This can be directly measured from the geodetic curvature and acceleration: the very few cases with a formally "good" curvature are due to the minimum for arc length and time span, thus presumably they contain very poor data with spurious curvature.

The Arcs of Type 4 and more provide "good" orbits in 97.8% of the cases. Actually, 86.7% have orbits with uncertainties 10 times less than the ones we have used to define "good" (see Figure 2, bottom right). Thus, this numerical test confirms the empirical guess that the critical values of the Arc Type which are most relevant for orbit determination[19] are 2 and 3.

The Arcs of Type 3 provide almost always some least squares orbit (in 99.1% of the cases; a full 6 parameter orbit in 96.9% of the cases): however, only 67.1% of the Type 3 Arcs have a "good" orbit. Of course this percentage is somewhat sensitive to the arbitrary controls we are using to define "good orbit": increasing by a factor 2 the standard deviations for (q, e, I) the percentage would increase to 78.4%. However, the qualitative result does not change: most Type 3 Arcs provide a useful orbit, good enough to discrimi-

[19] (Virtanen et al. 2005) describe this as one of the "phase transitions" of orbit determination.

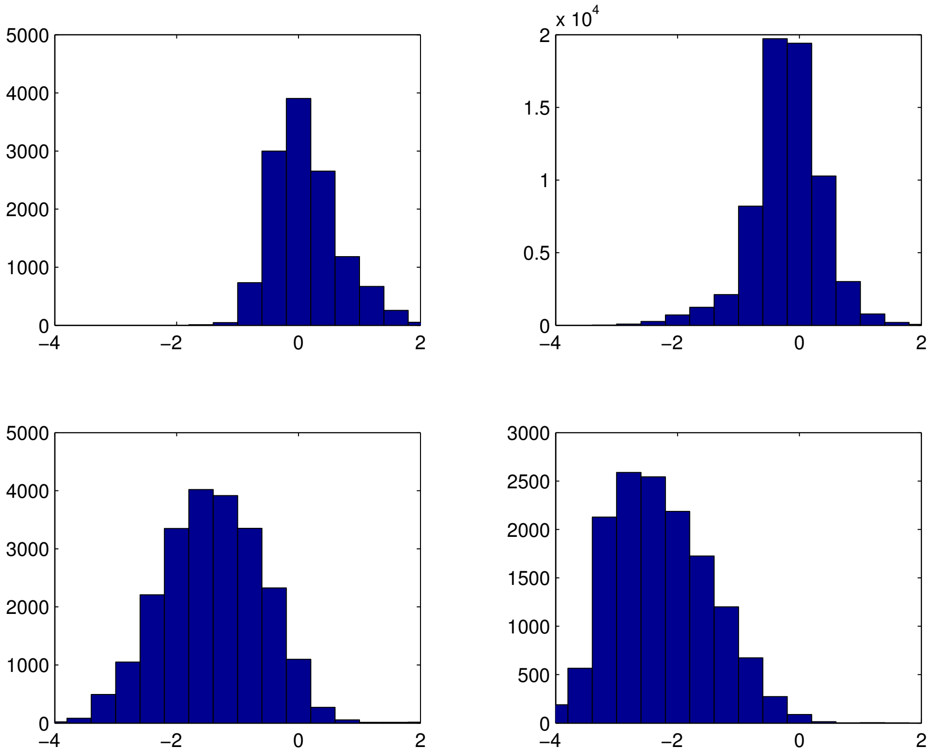

Fig. 2 Test with weak definition. Distribution of the standard deviations of the perihelion distance q (decimal logarithm in AU) for Observed Arcs of different Types. Top: Type 1 (left), Type 2 (right). Bottom: Type 3 (left), Type 4 (right).

nate between objects belonging to the main orbital classes, but by no means all of them (Figure 2, bottom left).

The Arcs of Type 2 provide "good" orbits only in 5.7% of the cases. Increasing the controls on the standard deviations by a factor 2 as above increases this percentage to 12.0% (Figure 2, top right). Again, the qualitative result is robust: the fact that an Observed Arc is of Type 2 is not enough to consider it provides a discovery. The question arises whether it would be possible to change the parameters, in particular χ^2_{min} which controls the "significance" of curvature, in such a way that an Arc of Type 2 would provide a good orbit. Some answer to this question is already contained in the Table: if we were to use $\chi^2_{min} = 100$ in the definition of significant curvature, the fraction of Type 2 Arcs with a "good" orbit would be 37.8%, and still the majority of them would not provide a useful orbit.

For the 104,289 Observed Arcs with a time span between the first and the last observation > 180 days our algorithm indicates that 93.1% have an Arc Type > 6; however, there is a small fraction (0.8%) of Type ≤ 4, and among those, 7.6% do not have a good orbit[20]. According to the definition, 8 of these long arcs should not be considered Discoveries, being of Type 2. In some of these cases the main problem is the reliability of the identification between two arcs in different apparitions.

Table 2 Arc Types (columns) and number of nights with observations (rows) for the test with $\chi^2 = 1$, $RMS_{min} = 3$.

Nights	Type 1	Type 2	Type 3	Type 4	Type>4	Total
1	3120	52	4	2	4	3182
2	14554	66088	3260	376	81	84359
3	20	6088	14610	1689	448	22855
4	4	373	4183	8888	1951	15399
>4	0	5	426	3255	39273	42959
Total	17698	72606	22483	14210	41757	168754

We need to comment on the relationship between number of observing nights and Arc Type (see Table 2). The data in the Table is mostly dominated by the principal diagonal, that is in most cases the number of nights and the Arc Type coincide. The main exception is that the Arcs of Type 1 are more often 2-nighters (82.2%). That is, the notion of Arc of Type 1 is very different from the one of 1-nighter.

For MBA with "good" orbit, 71.5% of Arcs of Type 3 have been observed over 3 nights and 65.5% of Arcs of Type 4 over 4 nights. The analogous argument for Type 2 Arcs does not work, because MBA with Arcs of Type 2 in most cases do not have a good orbit: in fact, of the Arcs of Type 2 with a "good" orbit, the overwhelming majority have 3 nights of observations.

For distant objects, the situation is totally different. If the orbit is good enough to decide that the object is indeed distant, but is not on a parabolic or hyperbolic orbit (say, $q - 3\, STD(q) > 10$ AU and $e + 3\, STD(e) < 1$), then the Arc Type must be ≥ 3 and the number of nights must be ≥ 4; also the arc time span needs to be > 50 days. The Type 2 Arcs, corresponding for distant objects to 3 or 4 nights, always have large uncertainties and do not allow us to discriminate TNO from objects belonging to different populations.

7.2 Test with strong definition of Arc Type

In a second test we have used the data disseminated by the MPC in March 2006, that is a larger dataset: 9.2 million observations. The Observed Arcs with a time span < 180 days were $185,926$: the increase is significant, indicating that more objects are "discovered", according to current MPC rules than the ones becoming multi-opposition and ultimately numbered.

We have used values $\chi^2_{min} = 9$ and $RMS_{min} = 4$, that is a comparatively *strong* definition of Arc Type, in which either a really significant curvature or a more conspicuous Z shape is required to split an Observed Arc.

[20] We even found 2 objects for which a least squares orbit could not be determined: this is a problem with the error model, with another weighting the orbits could be computed.

Table 3 The performance of the Observed Arc Types in orbit computation, test with strong definition of Arc Type and March 2006 data.

Arc Typ	Good orb curvature	Good orb no curv	Bad orb no curv	Bad orb curvature	Total
1	0	87	56,114	12	56,213
2	14,117	319	36,781	11,124	62,341
3	17,916	53	126	1,443	19,538
4	14,646	7	5	72	14,730
5	10,630	0	0	7	10,637
6	7,321	0	0	0	7,321
> 6	15,141	0	0	5	15,146

Of course the orbits computed from a given Arc Type are better determined when the definition is stronger (compare Figure 3 with Figure 2). However, the improvement is sharper for Types 3 and 4 (bottom panels). The Types 3 now have a "good" orbit in as much as 92% of the cases (Table 3). For Types 4 and 5 the number of cases with either no orbit or "bad" orbit is so small that they can be interpreted as strange cases, possibly with exceptionally low quality data.

The main change affecting Types 1 and 2 is the large increase in the number of Arcs rated as Type 1, as shown in Table 3. Nevertheless, the number of Type 1 Arcs with a "good" orbit has not significantly increased. This means that a large number of Arcs classified as Type 2 with the "weak" definition, but without a "good" orbit, have been reclassified as Type 1.

In conclusion, the results are sharper and the "strong" definition of Arc Type has better predictive value with respect to the accuracy of the orbit.

Table 4 Arc Types (columns) and number of nights with observations (rows) for the test with $\chi^2 = 9$, $RMS_{min} = 4$.

Nights	Type 1	Type 2	Type 3	Type 4	Type>4	Total
1	3482	20	5	3	3	3513
2	52599	41809	173	30	30	94641
3	123	17394	5593	125	117	23353
4	9	3082	9531	3065	250	15937
>4	0	36	250	11507	32704	48493
Total	56213	62341	19538	14730	33104	185926

The potential difficulty for the Observers with the "strong" definition is that it is not straightforward to predict the Arc Type for given number of nights of observations. The data in Table 4 are not at all dominated by the principal diagonal, that is there is not even statistically a simple correspondence between the number of nights and the Arc Type. For example, planning for 3 nights of observations does not guarantee in most cases a Dis-

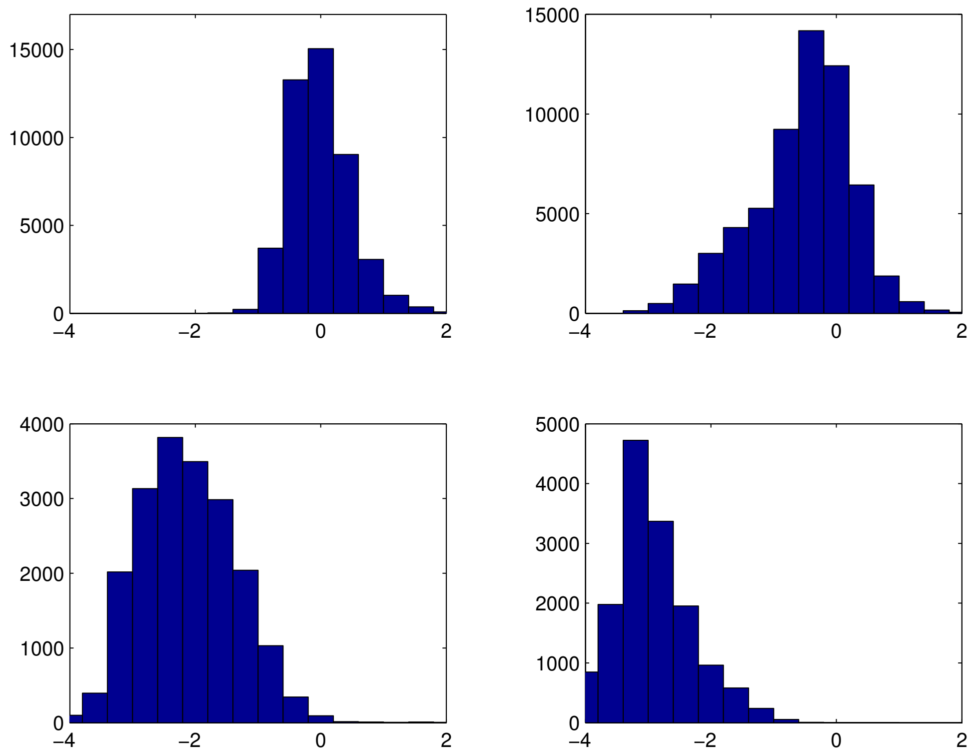

Fig. 3 Test with strong definition. Distribution of the standard deviations of the perihelion distance q (decimal logarithm in AU) for Observed Arcs of different Types. Top: Type 1 (left), Type 2 (right). Bottom: Type 3 (left), Type 4 (right).

covery. This does not mean that it is necessary to plan for 4 nights: at least at opposition, it is perfectly possible to obtain a Discovery level arc even with 3 nights, provided the spacing of the nights is uniform and the astrometric accuracy is high. The observation planning, however, is more complex and/or the requirements on data quality are stricter.

Table 4, when compared to Table 2, shows that in 5 months the number of single opposition designations has increased significantly, with more than half of this increase due to 2-nighters. This provides evidence supporting the statement made in Section 5 that the current definition of discovery (based upon 2 nights) does not anymore have positive effects on the observation scheduling, thus leaving a lot of asteroids "discovered" then lost.

8 Conclusions

We have proposed a definition of Discovery for a Solar System Object, covering essentially all the cases (with the exception of the natural satellites, for which the theory needs a significant adaptation). This definition has the advantage of being based upon a rigorously specified algorithm, depending only on few parameters; it can be applied in a fully automated way, thus it is objective and transparent.

The algorithm to compute the Arc Type has been programmed in the FORTRAN 95 language as an addition to the free software system *OrbFit*[21]; it is contained in the OrbFit distribution (versions 3.3.2 and later). With this software the Observers should also be able to compute Least Squares Orbits, although this is not necessary to get Discovery Credit according to the proposed rules.

Given the definition of Discovery, we have proposed rules to assign Credit, not only in the obvious case in which all the observations and the computations are submitted at once by a single author/group of authors, but also when different submissions have to concur to provide enough information to satisfy the definition of Discovery.

All the definitions we propose have been tested on the current public dataset of astrometric observations of non-numbered asteroids. The results of the test show that the definition of Discovery based upon Arcs of Type ≥ 3 is the best compromise between two requirements. The definition needs to be predictive of the quality and usefulness of the orbit which can be computed with the data, and needs to be simple and objective (not depending upon the skill of the Orbit Computer in actually squeezing an orbit out of the data).

The new definition depends upon four numerical parameters, namely: the minimum value of χ^2 for the curvature to be considered significant, the minimum value RMS_{min} of the (normalized) RMS with respect to a quadratic fit for the Z shape to be considered significant, the minimum arc time span, the minimum arc length as a curve on the celestial sphere. Thus, the definition of Discovery we are proposing can be adopted in different versions, depending upon the values selected for these parameters, especially χ^2_{min} and RMS_{min}. For example, we have tested two versions, a *weak* one and a *strong* one, with different comparative advantages. The weak version maintains in most cases the simple correspondence between Arc Type and number of nights of observations, thus is more straightforward to be used for observation scheduling. The strong version is better in ensuring that a Discovery corresponds to a knowledge of the Orbit sufficient to establish the nature of the object. On the other hand, the weak version produces results of inferior scientific value and/or requires more targeted follow up even after Discovery. The strong definition requires a more sophisticated observation planning and/or increased astrometric accuracy and/or more telescope time. From our experience primarily as Orbit Computers we are in favor of the strong version, because the results on the Orbits are much sharper, that is a Discovery corresponds in the overwhelming majority of cases to a useful orbit.

We have used in the tests an Error Model based on (Carpino et al. 2003): since the definition of Arc Type depends also upon the Error Model, a public Error Model is required to complete the definition of Discovery. This implies the need for a transition phase, in which an Error Model not fully satisfactory, even a very rough one, will have to be used. When a new and better Error Model will become available, by adopting it the Discovery definition is updated.

[21] The OrbFit distribution is available from http://newton.dm.unipi.it/orbfit/

A more complicated definition of Discovery, discriminating the cases in which even a Type 3 Arcs does in fact fail to provide a good orbit, and also reclassifying as discovery the small fraction of the Type 2 Arcs which indeed give a good orbit, could possibly be found. However, we do not see an easy way to give a definition with these additional requirements and which can still be computed by a simple algorithm. In our opinion the definition given here shows a good balance between simplicity and predictive value, thus we can recommend it for adoption by the IAU.

The new definition is a significant improvement with respect to the current practice, which results from the superposition of standards, rules and habits belonging to different epochs, characterized by asteroid searches performed with different technologies. We have taken into account the main lessons arising from the problems and controversies generated by the current practice. First, science is a cooperative process, thus it is normal that a discovery credit may be shared. Second, the rules must be transparent, not dependent upon discretionary choices and reproducible on everybody's computer. Third, the rules should not encourage keeping a discovery secret, although there cannot be retroactive punishment for secrecy, that is credit must be assigned based only on time of publication. Fourth, the rules must be appropriate for different orbital classes, even if this implies using a mathematical formalism more complicated than just counting nights.

We are not claiming our proposal solves all the problems and removes all controversies about discovery credit: this is simply not possible. The most critical problem is with TNOs. We have at least given a definition of Discovery implying that the Orbit is good enough to establish the nature of the object, while the current requirement of 2 nights was disastrously inadequate for this purpose. This leaves two problems unsolved. First, our proposal implies a fair allocation of Discovery Credit for the Orbit, but has no effect on the credit Observers may obtain for discoveries of other properties, such as satellites and/or exceptionally large sizes. Second, given that the timescale to accumulate enough Observations for Discovery may be months, even years, the Observers need either to keep the data for themselves and take the risk of being preceded in the announcement or make the data public and take the risk of having to share the Credit with others, who might precede them in the follow up.

The case of Comet Discovery has some additional complexity because it must account for observations both astrometric and proving the cometary activity. We have tried with our proposal to find an appropriate balance acknowledging all the contributions, but it is clear that on this point it is especially hard to achieve consensus.

We are in favor of open data policies and we would like to encourage early publication of data, but we do not think it would be acceptable to penalize those who choose not to do so. Thus we have proposed Rule 8, by which the data must be public at the time Discovery Credit is assigned, but not necessarily before.

The new definition is certainly more consistent and less prone to controversy; however, experience in using it may well dictate corrections and improvements. It is important that the definition is maintained consistent with logic, but also with the procedures really used in asteroid discoveries. The first of the next generation asteroid surveys should be operational soon: indeed this proposal has already taken into account the experience accumulated from the simulations of the Pan-STARRS survey, in which we have been involved (Milani et al. 2005b; Milani et al. 2006). The current practice will have to be changed as soon as some new generation survey is operational, on the contrary the new definition can work under the new conditions. Real operations being often quite different from what has been anticipated in the simulations, these definitions may need to be adapted, maybe just by changing some of the parameters and by developing a reliable Error Model, maybe in a more substantial way. The definitions and rules for credit must follow the progress of science, not the other way round.

ACKNOWLEDGMENTS

This research has been funded by the Italian *Ministero dell'Università e della Ricerca Scientifica e Tecnologica*, PRIN 2004 project "The Near Earth Objects as an opportunity to understand physical and dynamical properties of all the solar system small bodies" and by the *Ministry of Science and Environmental Protection of Serbia* project 146004 "Dynamics of Celestial Bodies Systems and Populations". We thank G.B. Valsecchi and R. Jedicke for enlightening discussions on this subject and S. Chesley for a careful review with many suggestions.

References

Boattini et al. 2006. Boattini, A., Milani, A., Gronchi, G.F., Valsecchi, G.B. 2006, in *Near Earth Objects, our Celestial Neighbors: Opportunity and Risk*, Milani et al. eds., submitted.

Carpino et al. 2003. Carpino, M., Milani, A., Chesley, S.R. 2003, Icarus, **166**, 248–270.

Danby 1985. Danby J.M.A. 1988, *Fundamentals of Celestial Mechanics*, Second edition, Willmann Bell, Richmond VA.

Milani et al. 2005a. Milani, A., Sansaturio, M.E., Tommei, G., Arratia, O., Chesley, S.R. 2005, Astron. Astrophys., **431**, 729–746.

Milani et al. 2005b. Milani, A., Gronchi, G.F., Knežević, Z., Sansaturio, M.E. and Arratia, O. 2005, Icarus, **79**, 350–374.

Milani et al. 2006. Milani A. et al. 2006, in *Asteroids Comets Meteors 2005*, D. Lazzaro et al., eds., Cambridge University Press, pp. 367–380.

Plummer 1918. Plummer, H.C. 1918 *An introductory treatise on dynamical astronomy*, Dover Publications, New York.

Sansaturio et al. 1996. Sansaturio, M.E., Milani, A. and Cattaneo, L. 1996, in *Dynamics, Ephemerides and Astrometry of the Solar System*, Ferraz-Mello, S. ed., Kluwer, pp. 193–198.

Virtanen et al. 2005. Virtanen, J, Muinonen, K., Granvik, M. and Laasko, T. 2005, in *Dynamics of Populations of Planetary Systems*, Knežević, Z. and Milani, A. eds., Cambridge University Press, pp. 239–248.